

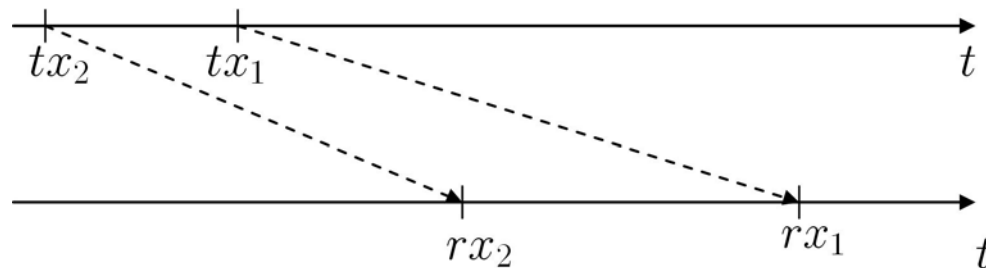
## Jitter and Some Math

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In the discussions of providing Voice and Video over IP packet data communications and services we inevitably consider Quality of Service aspects of both the media and the network behavior. From the media perspective factors such as packet rates and burstiness as well as the tolerances to impairments are always considered. For example, the characteristics and tolerances of VoIP streams are different from those of Video streams: VoIP has small packets, constant packet rate, higher (with respect to Video) tolerances to packet loss and packet jitter, Video on the other hand has larger packets, variable packet sizes and rates, low tolerance to packet loss and jitter. From the network perspective we always estimate if the network elements in the end-to-end path are capable of sustaining certain levels of performance parameters for packet rates, loss, jitter and delay.

The nature and impact of packet loss and delay is easy to understand and in most cases it is self explanatory. At the same time the nature of jitter and the quantification of jitter tolerances and Service Level Agreements are not as intuitive as for the packet loss and delay. For example, we often see parameters such as Average Jitter, Peak-to-Peak Jitter, and Instantaneous Jitter. What do they mean?

In the area of packet communications Jitter is referred to as Packet Delay Variation (PDV). It is the difference in the one-way end-to-end delay values for packets of a flow. Instantaneous PDV is the difference in packet transfer delays for successive packets – this is what is usually called Jitter. Often Jitter is measured in terms of a time deviation from the nominal packet inter-arrival times for successive packets. Consider a media stream with a constant packet rate (such as VoIP where packets are sent every 20 milliseconds). Figure below shows two successive packets in time:



Where:

$tx_1$  - Moment in time the first packet was transmitted

$tx_2$  - Moment in time the second packet was transmitted

$rx_1$  - Moment in time the first packet was received

$rx_2$  - Moment in time the second packet was received

Then:

$t_{d_1} = (rx_1 - tx_1)$  is the transfer delay of the first packet

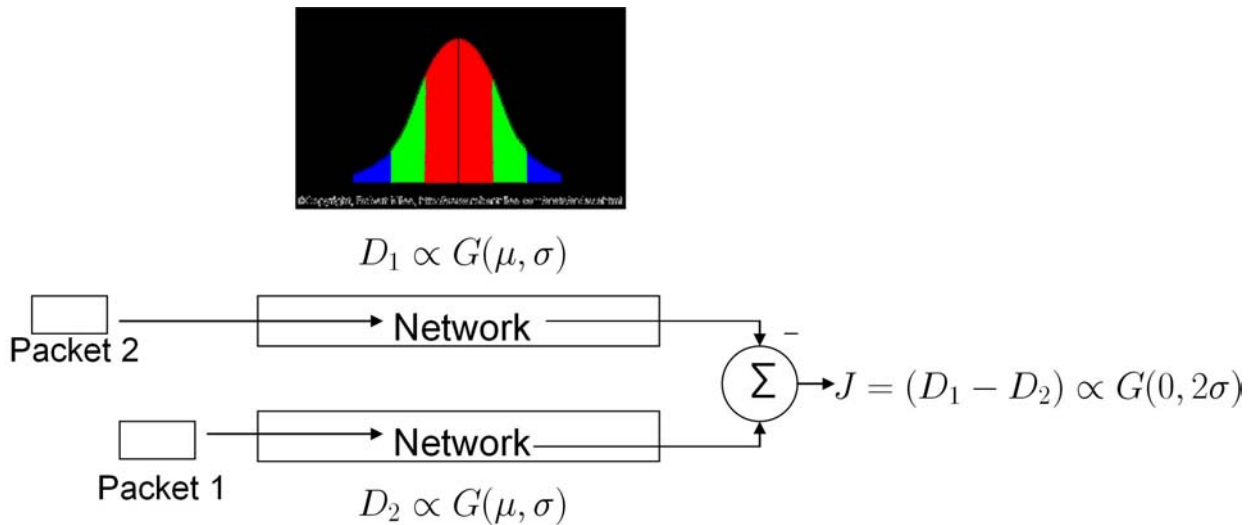
$t_{d_2} = (rx_2 - tx_2)$  is the transfer delay of the second packet

$J = (t_{d_1} - t_{d_2}) = [(rx_1 - rx_2) - (tx_1 - tx_2)]$  is the value of the Instantaneous Jitter.

Note that  $(tx_1 - tx_2)$  is the nominal inter-packet gap (20 milliseconds in the VoIP example) and  $(rx_1 - rx_2)$  is the actual packet inter-arrival time. The difference between the two is the Instantaneous Jitter which is the deviation of the actual packet inter-arrival times from the

nominal inter-packet gap. Also note that  $J$  may be positive (packet dispersion) or negative (packet clamping).

Now consider a network path. From the statistical point of view the delay performance of a network path may be captured in a Gaussian (bell curve) distribution of one-way delay with a given mean value  $\mu$  and a given standard deviation  $\sigma$ . In other words, on average the network path will cause delay of  $\mu$  milliseconds, and in 99% of cases the delay will vary in the interval  $[\mu - 3\sigma, \mu + 3\sigma]$ . Let  $D \propto G(\mu, \sigma)$  be a random variable that captures the one-way delay of the network path. When successive packets traverse the path with delay  $D$ , Jitter  $J$  may be expressed as a difference of two identically distributed Gaussian random variables. This is shown in figure below:



Jitter as a difference of two identically distributed Gaussian random variables is also a Gaussian random variable with mean equal to the difference of means and the standard deviation equal to the sum of standard deviations of the original delay random variables. Note that since the original random variables are identically distributed the mean of the Jitter random variable is 0 and its standard deviation is twice that of the delay. In other words the Average Jitter is 0, and therefore we need to consider the standard deviation of the variation in the transfer delay when we quantify Jitter.

The standard deviation of the network path delay  $\sigma$  may be used to define a measure of Jitter that is called a “Peak-to-Peak Jitter”. Given a one-way delay standard deviation of  $\sigma$  milliseconds we can consider a Peak-to-Peak Jitter interval of  $[-2\sigma, +2\sigma]$  and a Peak-to-Peak Jitter of  $4\sigma$ . For example, if a network segment has an average delay of 100 milliseconds and a delay standard deviation of 5 milliseconds, the Peak-to-Peak Jitter of 20 milliseconds may be considered.